Conditional Statements

If. . . then. . .

Given statements P and Q, we can form a new statement "If P, then Q".

- If (it is raining), then (it is cloudy).
- If (a number is divisible by 6), then (it is even).

These are called **conditional statements.**

Symbolically

$$P \implies Q$$

.

If. . . then..

Truth of the first statement forces truth of the second statement.

$P \Rightarrow Q$

TRUE

FALSE

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Alternatives

- If P, then Q
- Q if P
- Q whenever P
- Q, provided that P
- Whenever P, then also Q
- P is a sufficient condition for Q

more alternatives

- For Q, it is sufficient that P
- Q is a necessary condition for P
- For P, it is necessary that Q
- P only if Q

Example

A matrix is invertible provided that its determinant is non-zero.

P det $\neq 0$

Q invertible

$$P \Rightarrow Q$$

Example

An integer is divisible by 8 only if it is divisible by 4.

Example

Being a native born citizen over the age of 35 is sufficient to be eligible to be elected president of the US.

P native born

Q ≥ 35

R eligible

P and Q $\Rightarrow$ R

Example

Being over the age of 35 is necessary to be elected president of the US.

Statements

Open Sentences $P(x)$: "x is an even number"

$P, Q \rightsquigarrow$ P and Q $(P \wedge Q)$

$\rightsquigarrow$ P or Q $(P \vee Q)$

not P $\neg P$ $\sim P$

$P \Rightarrow Q$ ~~means~~ is true if P is true and Q is true or P is false

if P then Q

$P \Rightarrow Q$ (P and Q) or $\neg P$

P	Q	P and Q	$\neg P$	(P and Q) or $\neg P$
T	T	T	F	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

If its raining then its cloudy $P \Rightarrow Q$

(its raining and cloudy) or its not raining

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$\boxed{P \Rightarrow Q \text{ means that } Q \Rightarrow P}$

converse

TRUE

$P \Rightarrow Q$ same as $\neg Q \Rightarrow \neg P$

contrapositive

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T